Answers

Problem 1 Median filtering (20 pt)

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

$\mathcal{M}(f_1)$

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

$\mathcal{M}(f_2)$

a. (5pt) Pictures of the median-filtered images $\mathcal{M}(f_1)$ and $\mathcal{M}(f_2)$ are shown in the figure above. Only the center pixels of the output images have a non-zero value. For example, for image f_1, the 9 values around the center pixel are: 10, 9, 0, 5, 0, 4, 0, 1, 0. After sorting these in increasing order, we get: 0, 0, 0, 0, 1, 4, 5, 9, 10. The middle value is 1, so the value of $\mathcal{M}(f_1)$ at the center pixel is 1. If the 3×3 mask is centered on any other pixel p in the input image, there are at least 5 pixels with value zero within the mask, so the middle value of the 9 ordered values is 0, hence median value is zero as well. For f_2 the reasoning is analogous.

b. (5pt) The sum image $f_1 + f_2$ and the output of the median filter $\mathcal{M}(f_1 + f_2)$ are as follows:

0	0	0	0	0
0	0	15	0	0
0	16	11	12	0
0	0	14	0	0
0	0	0	0	0

$f_1 + f_2$

0	0	0	0	0
0	0	0	0	0
0	0	11	0	0
0	0	0	0	0
0	0	0	0	0

$\mathcal{M}(f_1 + f_2)$

c. (5pt) For an image filter $\mathcal{O}$ to be linear, the identity $\mathcal{O}(f_1 + f_2) = \mathcal{O}(f_1) + \mathcal{O}(f_2)$ has to be satisfied. As you can see from the above results, the center pixel in the image $\mathcal{M}(f_1 + f_2)$ has the value 11, while the sum of the outputs, i.e. $\mathcal{M}(f_1) + \mathcal{M}(f_2)$, has the value 2 at the center pixel. So the median filter is not a linear image operation.

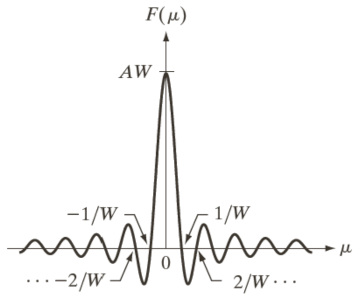
d. (5pt) The median filter is shift-invariant, because the same mask is moved to all image pixels. If the input image f is shifted, then the output image $\mathcal{M}(f)$ will be shifted by the same amount. Since the median filter is not linear, it cannot be represented as a convolution filter.

Problem 2 Frequency domain filtering (20 pt)

a. (5pt) The purpose of lowpass filtering is to remove high-frequency structures in the image.

b. (5pt) ILPF filtering causes: (i) blurring: the image becomes more fuzzy, sharp details are lost; and (ii) ringing artefacts, as for example is visible around the letter "a" in the filtered images.

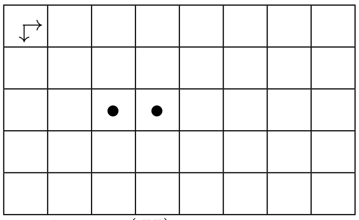
c. (5pt) The spatial representation $h(x)$ of a one-dimensional ILPF is a sinc function. In the spatial domain, the effect of the ILPF filter can be described as a convolution with the sinc function $h(x)$, see the following picture.



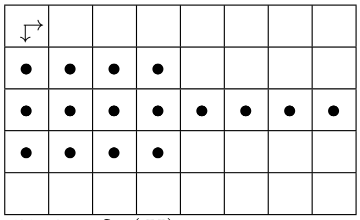
The center lobe of the sinc is the main cause of the blurring, while the outer, smaller lobes are mainly responsible for ringing.

d. (5pt) The more the cut-off radius D_0 is increased, the more the sinc function approaches an impulse, so blurring and ringing will decrease.

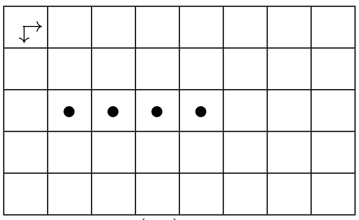
Problem 3 Morphological filtering (20 pt)



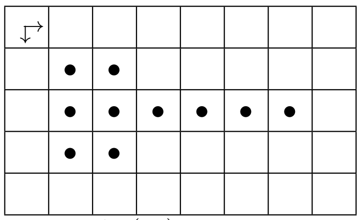
erosion $\varepsilon_B(X)$



dilation $\delta_B(X)$



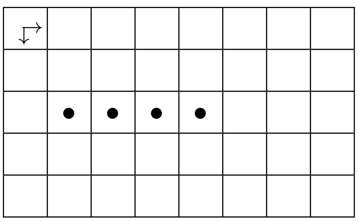
opening $\gamma_B(X)$



closing $\phi_B(X)$

a. (10pt) See the figure above.

b. (10pt) The result of the opening by reconstruction is in the figure below. The opening of the input X contains the parts of X where the structuring element fits completely, so the opening can have 1-pixels in different connected components. But in the opening by reconstruction there are only 1-pixels in the connected component of X containing the marker. In this example the results of the opening and the opening by reconstruction are the same, but in general this will not be the case.



Problem 4 Image segmentation (15 pt)

a. (5pt) Image segmentation is the process of partitioning an image into non-overlapping regions, such that each region is *uniform* with respect to some property, but the union of adjacent regions is not uniform with respect to the same property.

b. (5pt) Edge-based segmentation is based on some form of differentiation of the input image. In the case of noise in the input image, this differentiation process leads to very unstable results. This can be improved by first processing the image by some noise-reducing filter which smooths the image, such as Gaussian convolution.

c. (5pt) In the split-and-merge method, we start with the entire image as the initial region. Then we split regions successively into subregions if a region does not satisfy a homogeneity predicate Q. Next we merge adjacent regions when their union satisfies Q. Then we repeat the whole process until no more merging takes place.

Problem 5 Image description (15 pt)

a. (5pt) We can use the position operator $Q(p, q)$: "q is $2m$ pixels to the right and $2m$ pixels below with respect to p". In that case $Q(p, q)$ is only satisfied when both pixels in a pair have the same value (both black or both white). So the co-occurrence matrix will be diagonal.

b. (5pt) The image has coding redundancy: 1 bit is sufficent to encode a black and white image. The image also has spatial redundacy, since there is a regular spatial pattern of black and white squares.

c. (5pt) Reducing coding redundancy: e.g., by Huffman coding. Reducing spatial redundancy: e.g., by run-length coding or transform coding.